The electric dipole moment of the neutron in chiral perturbation theory [1]

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Abstract

We calculate the electric dipole moments of the neutron and the Λ within the framework of heavy baryon chiral perturbation theory. They are induced by strong CP-violating terms of the effective Lagrangian in the presence of the vacuum angle θ_0. The construction of such a Lagrangian is outlined and we are able to give an estimate for θ_0.

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1 Introduction

The axial $U(1)$ anomaly in QCD implies an additional term in the Lagrangian, which violates P, T and CP. This new term is proportional to the so-called vacuum angle θ_0, an unknown parameter, and its size may be determined from CP-violating effects, as e.g. $\eta \to \pi\pi$ or the electric dipole moments of the neutron and the Λ. The most recent measurements of the electric dipole moment of the neutron d_n^γ have constrained it to [1]

$$|d_n^\gamma| < 6.3 \times 10^{-26}\, e\,\mathrm{cm}\,. \tag{1}$$

On the other hand, theoretical estimates for d_n^γ induced by the θ_0-term can be given leading to an upper bound for θ_0 [2, 3, 4]. Of particular interest here is the estimate of Pich and de Rafael [3] who used an effective chiral Lagrangian approach and came to the conclusion that it is possible to obtain an estimate for the size of the vacuum angle θ_0 with an experimental upper limit of $|\theta_0| \leq 5 \times 10^{-10}$. However, the authors worked in a relativistic framework which does not have a systematic chiral counting scheme, so that higher loop diagrams contribute to lower chiral orders. This problem is avoided in heavy baryon chiral perturbation theory as proposed in [5], which allows for a consistent power counting. One should therefore study this effect within the heavy baryon formulation. Furthermore, the baryon Lagrangian in [3] which describes the interactions of the neutron with the pseudoscalar nonet (π, K, η, η') does not contain explicitly CP-violating terms. They are rather induced by the vacuum alignment of the purely mesonic Lagrangian in the presence of the θ_0-term. As shown in [6, 7] the most general baryonic Lagrangian taking the axial $U(1)$ anomaly into account does have explicitly CP-violating terms even at lower chiral orders. It has to be checked, if such terms lead to sizeable contributions for the present consideration. Finally, the authors of [3] proposed to estimate the contribution from unknown counterterms by varying the scale in the chiral logarithms. This procedure reveals the scale dependence of the involved coupling constants but not their absolute value, and it is desirable to have a somewhat more reliable estimate of the involved couplings. The aim of the present work is to reinvestigate the electric dipole moments of the neutron and the Λ by taking the above mentioned points into consideration. One has to check, if it is still possible to give a reliable estimate of the vacuum angle θ_0, and if it is different from the one given in [3].

The paper is organized as follows. In the next section we present the purely mesonic effective Lagrangian in the presence of the θ_0-term and the vacuum alignment is discussed. Baryon fields are included in the effective theory in Sec. 3 by using the method outlined in [6]. We proceed by calculating in Sec. 4 the electric dipole moment of the neutron and the Λ up to one-loop order within the heavy baryon framework. Numerical results and conclusions are given in Sec. 5.

2 The mesonic Lagrangian

In this section, we will consider the purely mesonic Lagrangian in the presence of the θ_0-term. The derivation of this Lagrangian has been given elsewhere, see e.g. [8, 9, 10], so we will restrict ourselves to the repetition of some of the basic formulae which are needed in the present work. In [9, 10] the topological charge operator coupled to an external field is added to the QCD Lagrangian

$$\mathcal{L} = \mathcal{L}_{QCD} - \frac{g^2}{16\pi^2}\theta(x)\mathrm{tr}_c(G_{\mu\nu}\tilde{G}^{\mu\nu}) \tag{2}$$

with $\tilde{G}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ and tr_c is the trace over the color indices. Under $U(1)_R \times U(1)_L$ the axial $U(1)$ anomaly adds a term $-(g^2/16\pi^2) 2N_f\, \alpha\, \mathrm{tr}_c(G_{\mu\nu}\tilde{G}^{\mu\nu})$ to the QCD Lagrangian, with N_f being the number of different quark flavors and α the angle of the global axial $U(1)$ rotation. The vacuum angle $\theta(x)$ is in this context treated as an external field that transforms under an axial $U(1)$ rotation as

$$\theta(x) \rightarrow \theta'(x) = \theta(x) - 2N_f\alpha. \tag{3}$$

Then the term generated by the anomaly in the fermion determinant is compensated by the shift in the θ source and the Lagrangian from Eq. (2) remains invariant under axial $U(1)$ transformations. The symmetry group $SU(3)_R \times SU(3)_L$ of the Lagrangian $\mathcal{L}_{QCD}$ is extended[3] to $U(3)_R \times U(3)_L$ for $\mathcal{L}$. The Green functions of QCD are obtained by expanding the generating functional around $\theta(x) = \theta_0$ where the phase of the quark mass matrix emerging from the Yukawa couplings of the light quarks in the electroweak sector has been absorbed in θ_0. The extended symmetry remains at the level of an effective theory and the additional source θ also shows up in the effective Lagrangian. Let us consider the purely mesonic effective theory first. The lowest lying pseudoscalar meson nonet is summarized in a matrix valued field $\tilde{U}(x)$. The effective Lagrangian is formed with the fields $\tilde{U}(x)$, derivatives thereof and also includes both the quark mass matrix $\mathcal{M}$ and the vacuum angle θ: $\mathcal{L}_{\mathrm{eff}}(\tilde{U}, \partial\tilde{U}, \ldots, \mathcal{M}, \theta)$. Under $U(3)_R \times U(3)_L$ the fields transform as follows:

$$\tilde{U}' = R\tilde{U}L^\dagger \quad , \qquad \mathcal{M}' = R\mathcal{M}L^\dagger \quad , \qquad \theta'(x) = \theta(x) - 2N_f\alpha \tag{4}$$

with $R \in U(3)_R$, $L \in U(3)_L$, but the Lagrangian remains invariant. The phase of the determinant of $\tilde{U}(x)$ transforms under axial $U(1)$ as $\ln\det\tilde{U}'(x) = \ln\det\tilde{U}(x) + 2iN_f\alpha$ so that the combination $\theta - i\ln\det\tilde{U}$ remains invariant. It is more convenient to replace the variable θ by this invariant combination, $\mathcal{L}_{\mathrm{eff}}(\tilde{U}, \partial\tilde{U}, \ldots, \mathcal{M}, \theta - i\ln\det\tilde{U})$. One can now construct the effective Lagrangian in these fields that respects the symmetries of the underlying theory. In particular, the Lagrangian is invariant under $U(3)_R \times U(3)_L$ rotations of $\tilde{U}$ and $\mathcal{M}$ at a fixed value of the last argument. The Lagrangian up to and including terms with two derivatives and one factor of $\mathcal{M}$ reads

$$\begin{aligned}\mathcal{L}_\phi \;=\; & -V_0 + V_1\langle \nabla_\mu \tilde{U}^\dagger \nabla^\mu \tilde{U}\rangle + V_2\langle \tilde{\chi}^\dagger \tilde{U} + \tilde{\chi}\tilde{U}^\dagger\rangle + iV_3\langle \tilde{\chi}^\dagger \tilde{U} - \tilde{\chi}\tilde{U}^\dagger\rangle \\ & + V_4\langle \tilde{U}\nabla_\mu \tilde{U}^\dagger\rangle\langle \tilde{U}^\dagger \nabla^\mu \tilde{U}\rangle.\end{aligned} \tag{5}$$

The expression $\langle \ldots \rangle$ denotes the trace in flavor space and $\tilde{\chi}$ is proportional to the quark mass matrix $\tilde{\chi} = \tilde{\chi}^\dagger = 2B_0\mathcal{M}$ with $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ and $B_0 = -\langle 0|\bar{q}q|0\rangle/F_\pi^2$ the order parameter of the spontaneous symmetry violation. The covariant derivative of $\tilde{U}$ is defined by

$$\nabla_\mu \tilde{U} = \partial_\mu \tilde{U} - i(v_\mu + a_\mu)\tilde{U} + i\tilde{U}(v_\mu - a_\mu). \tag{6}$$

The external fields $v_\mu(x), a_\mu(x)$ represent Hermitian 3×3 matrices in flavor space. Note that a term of the type $iV_5\langle \tilde{U}^\dagger \nabla_\mu \tilde{U}\rangle \nabla^\mu\theta$ can be transformed away [9] and a term proportional to $V_6 \nabla_\mu\theta\nabla^\mu\theta$ does not enter the calculations performed in the present work and will be neglected. The coefficients V_i are functions of the variable $\theta - i\ln\det\tilde{U}$, $V_i(\theta - i\ln\det\tilde{U})$, and can be expanded in terms of this variable. The terms $V_{1,\ldots,4}$ are of second chiral order, whereas V_0 is of zeroth chiral order. Parity conservation implies that the V_i are all even functions of

[3]To be more precise, the Lagrangian changes by a total derivative which gives rise to the Wess-Zumino term. We will neglect this contribution since the corresponding terms involve five or more meson fields which do not play any role for the discussions here.

$\theta - i \ln \det\tilde{U}$ except V_3, which is odd, and $V_1(0) = V_2(0) = F_\pi^2/4$ gives the correct normalizaton for the quadratic terms of the Goldstone boson octet, where $F_\pi \simeq 92.4$ MeV is the pion decay constant.

In order to use the effective Lagrangian, one must first determine the vacuum expectation value of $\tilde{U}$ by minimizing the potential energy

$$V(\tilde{U}) = V_0 - V_2\langle\tilde{\chi}^\dagger\tilde{U} + \tilde{\chi}\tilde{U}^\dagger\rangle - iV_3\langle\tilde{\chi}^\dagger\tilde{U} - \tilde{\chi}\tilde{U}^\dagger\rangle. \tag{7}$$

Since $\tilde{\chi}$ is diagonal, one can assume the minimum U_0 to be diagonal as well and of the form

$$U_0 = \mathrm{diag}(\mathrm{e}^{-i\varphi_u}, \mathrm{e}^{-i\varphi_d}, \mathrm{e}^{-i\varphi_s}). \tag{8}$$

In terms of the angles φ_q the potential becomes

$$V(U_0) = V_0(\bar{\theta}_0) - 4\,B_0\,V_2(\bar{\theta}_0)\sum_q m_q \cos\varphi_q - 4\,B_0\,V_3(\bar{\theta}_0)\sum_q m_q \sin\varphi_q, \tag{9}$$

where we have introduced the notation $\bar{\theta}_0 = \theta_0 - \sum_q \varphi_q$. The Taylor expansions of the functions V_i read

$$\begin{aligned} V_i(\bar{\theta}_0) &= \sum_{n=0}^{\infty} V_i^{(2n)}\bar{\theta}_0^{2n} \qquad\qquad \text{for } i = 0, 2 \\ V_3(\bar{\theta}_0) &= \sum_{n=0}^{\infty} V_i^{(2n+1)}\bar{\theta}_0^{2n+1} \end{aligned} \tag{10}$$

with coefficients not fixed by chiral symmetry. Minimizing the potential with respect to the angles φ_q leads to

$$2B_0\, m_q\, \sin\varphi_q = \mathcal{A} + 2B_0\,\mathcal{B}\, m_q\, \cos\varphi_q \tag{11}$$

with

$$\begin{aligned} \mathcal{A} \;=\; & 2B_0\Big(\sum_{n=0}^{\infty} V_2^{(2n)}\,\bar{\theta}_0^{2n}\Big)^{-1} \quad \sum_{n=1}^{\infty}\Big(\frac{1}{2B_0}n\,V_0^{(2n)}\,\bar{\theta}_0^{2n-1} \\ & -2n\,V_2^{(2n)}\,\bar{\theta}_0^{2n-1}\sum_{j=u,d,s} m_j\cos\varphi_j \\ & -(2n-1)V_3^{(2n-1)}\bar{\theta}_0^{2n-2}\sum_{j=u,d,s} m_j \sin\varphi_j\Big) \end{aligned} \tag{12}$$

and

$$\mathcal{B} = \Big(\sum_{n=0}^{\infty} V_2^{(2n)}\,\bar{\theta}_0^{2n}\Big)^{-1}\sum_{n=0}^{\infty} V_3^{(2n+1)}\,\bar{\theta}_0^{2n+1}. \tag{13}$$

To lowest order both in the quark masses m_q and $1/N_c$ Eq. (11) reads

$$\frac{1}{2}B_0\,F_\pi^2\,m_q\sin\varphi_q = V_0^{(2)}\,\bar{\theta}_0 \tag{14}$$

which is the equation for the φ_q considered in [3, 11]. One then writes

$$\tilde{U} = \sqrt{U_0}\,U\,\sqrt{U_0} \tag{15}$$

and U can be parametrized as

$$U(\phi, \eta_0) = \exp\{2i\phi/F_\pi + i\sqrt{\frac{2}{3}}\eta_0/F_0\}, \tag{16}$$

where the singlet η_0 couples to the singlet axial current with strength F_0. The unimodular part of the field $U(x)$ contains the degrees of freedom of the Goldstone boson octet ϕ

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} , \tag{17}$$

while the phase $\det U(x) = e^{i\sqrt{6}\eta_0/F_0}$ describes the η_0 . The diagonal subgroup $U(3)_V$ of $U(3)_R \times U(3)_L$ does not have a dimension-nine irreducible representation and consequently does not exhibit a nonet symmetry. We have therefore used the different notation F_0 for the decay constant of the singlet field.

One can now express the effective Lagrangian in terms of the Goldstone boson matrix U and the angles φ_q

$$\begin{aligned} \mathcal{L}_\phi &= -V_0 + V_1\langle\nabla_\mu U^\dagger \nabla^\mu U\rangle + [V_2 + \mathcal{B}V_3]\langle\chi(U + U^\dagger)\rangle - i\mathcal{A}V_2\langle U - U^\dagger\rangle \\ &+ i[V_3 - \mathcal{B}V_2]\langle\chi(U - U^\dagger)\rangle + \mathcal{A}V_3\langle U + U^\dagger\rangle + V_4\langle U\nabla_\mu U^\dagger\rangle\langle U^\dagger\nabla^\mu U\rangle, \end{aligned} \tag{18}$$

where we have absorbed the angles φ_q in hermitian matrices χ and H by defining

$$\sqrt{U_0^\dagger}\,\tilde{\chi}\,\sqrt{U_0^\dagger} = \chi + iH, \qquad\qquad \sqrt{U_0}\,\tilde{\chi}^\dagger\,\sqrt{U_0} = \chi - iH, \tag{19}$$

so that $\chi = 2B_0\mathrm{diag}(m_q\cos\varphi_q)$ and $H = 2B_0\mathrm{diag}(m_q\sin\varphi_q) = \mathcal{A}+\mathcal{B}\chi$. The V_i are functions of $\sqrt{6}\eta_0/F_0+\bar{\theta}_0$, $V_i(\sqrt{6}\eta_0/F_0+\bar{\theta}_0)$ and we have assumed the external fields v_μ and a_μ to be diagonal which is the case if one considers electromagnetic interactions. Note that the Goldstone boson masses at lowest chiral order are not only functions of the current quark masses m_q, but also depend on the angles $\cos\varphi_q$. The kinetic energy of the η_0 singlet field obtains contributions from $V_1\langle\nabla_\mu U^\dagger\nabla^\mu U\rangle$ and $V_4\langle U\nabla_\mu U^\dagger\rangle\langle U^\dagger\nabla^\mu U\rangle$ which read

$$\Big(\frac{F_\pi^2}{2F_0^2} + \frac{6}{F_0^2}V_4(0)\Big)\partial_\mu\eta_0\partial^\mu\eta_0. \tag{20}$$

We renormalize the η_0 field in such a way that the coefficient in brackets is $1/2$ in analogy to the kinetic term of the octet. By redefining F_0 and keeping for simplicity the same notation both for η_0 and F_0 one arrives at the same Lagrangian as in Eq. (18) but with $V_4(0) = (F_0^2 - F_\pi^2)/12$ in order to ensure the usual normalization for the kinetic term of a pseudoscalar particle.

3 CP-violating terms in the baryon Lagrangian

As mentioned before, another source for CP-violation is the baryon Lagrangian. The CP-violating terms can be divided into two groups. Firstly, the vacuum alignment of the mesonic Lagrangian induces CP non-conserving meson-baryon interactions as considered in [3]. But secondly, there are also explicitly CP-violating terms in the most general Lagrangian in the presence of the θ-vacuum angle which have been neglected in [3]. In order to construct the

Lagrangian in the baryon-sector, one has to adopt a non-linear representations for baryons. The main ingredient for a non-linear realization is the compensator field $K(\tilde{U}, R, L) \in U(3)_V$, which appears in the chiral $U(3)_L \times U(3)_R$ transformation of the left and right coset representatives, $\tilde{\xi}_L(\tilde{U})$ and $\tilde{\xi}_R(\tilde{U})$:

$$\begin{aligned} \tilde{\xi}_L(\tilde{U}) &\rightarrow L\,\tilde{\xi}_L(\tilde{U})\,K^\dagger(\tilde{U}, R, L) \\ \tilde{\xi}_R(\tilde{U}) &\rightarrow R\,\tilde{\xi}_R(\tilde{U})\,K^\dagger(\tilde{U}, R, L). \end{aligned} \tag{21}$$

The field $\tilde{U}$ from the last section is defined as

$$\tilde{U} = \tilde{\xi}_R\,\tilde{\xi}_L^\dagger. \tag{22}$$

The baryon octet B is given by the matrix

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \tag{23}$$

which transforms as a matter field

$$B \rightarrow B' = KBK^\dagger. \tag{24}$$

The covariant derivative of the baryon fields reads

$$[\tilde{D}_\mu, B] = \partial_\mu B + [\tilde{\Gamma}_\mu, B] \tag{25}$$

with $\tilde{\Gamma}_\mu$ being the chiral connection

$$\tilde{\Gamma}_\mu = \frac{1}{2}\Big[\tilde{\xi}_R^\dagger\,(\partial_\mu - ir_\mu)\,\tilde{\xi}_R \;+\; \tilde{\xi}_L^\dagger\,(\partial_\mu - il_\mu)\,\tilde{\xi}_L\Big] \tag{26}$$

and $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$. For electromagnetic interactions the external fields are $a_\mu = 0$ and $v_\mu = -eQ\mathcal{A}_\mu$ with the quark charge matrix $Q = \frac{1}{3}\mathrm{diag}(2, -1, -1)$. In order to incorporate the interactions with the mesons into the effective theory it is convenient to form an object of axial-vector type with one derivative

$$\tilde{\xi}_\mu = i\Big[\tilde{\xi}_R^\dagger\,(\partial_\mu - ir_\mu)\,\tilde{\xi}_R \;-\; \tilde{\xi}_L^\dagger\,(\partial_\mu - il_\mu)\,\tilde{\xi}_L\Big]. \tag{27}$$

Further ingredients of the non-linear representation are

$$\tilde{\chi}_\pm = \tilde{\xi}_L^\dagger\,\tilde{\chi}^\dagger\,\tilde{\xi}_R \pm \tilde{\xi}_R^\dagger\,\tilde{\chi}\,\tilde{\xi}_L \tag{28}$$

and the quantity

$$\tilde{F}^\pm_{\mu\nu} = \tilde{\xi}_R^\dagger\,F^R_{\mu\nu}\,\tilde{\xi}_R \pm \tilde{\xi}_L^\dagger\,F^L_{\mu\nu}\,\tilde{\xi}_L, \tag{29}$$

where $F^{R/L}_{\mu\nu}$ are the field strength tensors of r_μ/l_μ. The most general relativistic effective Lagrangian up to second order in the derivative expansion and contributing to the electric dipole moments of the neutron and Λ reads

$$\begin{aligned} \mathcal{L}_{\phi B} = \; & iW_1\langle[\tilde{D}^\mu, \bar{B}]\gamma_\mu B\rangle - iW_1^*\langle\bar{B}\gamma_\mu[\tilde{D}^\mu, B]\rangle + W_2\langle\bar{B}B\rangle \\ & + W_3\langle\bar{B}\gamma_\mu\gamma_5\{\tilde{\xi}^\mu, B\}\rangle + W_4\langle\bar{B}\gamma_\mu\gamma_5[\tilde{\xi}^\mu, B]\rangle + W_5\langle\bar{B}\gamma_\mu\gamma_5 B\rangle\langle\tilde{\xi}^\mu\rangle \\ & + iW_6\langle\bar{B}\gamma_5 B\rangle + W_7\langle\bar{B}\{\tilde{\chi}_+, B\}\rangle + W_8\langle\bar{B}[\tilde{\chi}_+, B]\rangle + W_9\langle\bar{B}B\rangle\langle\tilde{\chi}_+\rangle \\ & + iW_{10}\langle\bar{B}\{\tilde{\chi}_-, B\}\rangle + iW_{11}\langle\bar{B}[\tilde{\chi}_-, B]\rangle + iW_{12}\langle\bar{B}B\rangle\langle\tilde{\chi}_-\rangle \\ & + iW_{13}\langle\bar{B}\sigma_{\mu\nu}\gamma_5\{\tilde{F}_+^{\mu\nu}, B\}\rangle + iW_{14}\langle\bar{B}\sigma_{\mu\nu}\gamma_5[\tilde{F}_+^{\mu\nu}, B]\rangle \\ & + iW_{15}\langle\bar{B}\sigma_{\mu\nu}\gamma_5 B\rangle\langle\tilde{F}_+^{\mu\nu}\rangle. \end{aligned} \tag{30}$$

The W_i are functions of the combination $\sqrt{6}\eta_0/F_0 + \bar{\theta}_0$. From parity it follows that $W_{1,\dots,5}$ and $W_{7,8,9}$ are even in this variable, whereas W_6 and $W_{10,\dots,15}$ are odd. The latter have not been taken into account in [3]. One can further reduce the number of independent terms by making the following transformation. By decomposing the baryon fields into their left- and right handed components

$$B_{R/L} = \frac{1}{2}(1 \pm \gamma_5)B \tag{31}$$

and transforming the left- and right-handed states separately via

$$\begin{aligned} B_{R/L} &\rightarrow \frac{1}{\sqrt{W_2 \pm iW_6}} B_{R/L} \\ \bar{B}_{R/L} &\rightarrow \frac{1}{\sqrt{W_2 \mp iW_6}} \bar{B}_{R/L} \end{aligned} \tag{32}$$

one can eliminate the $\langle \bar{B}\gamma_5 B\rangle$ term and simplify the coefficient of $\langle \bar{B}B\rangle$. The details of this calculation are given in [6]. Note that this transformation leads to mixing of the terms of the type $\bar{B}\tilde{\chi}_\pm B$ with $\bar{B}\gamma_5\tilde{\chi}_\pm B$ which are of third chiral order and have been neglected here. Furthermore, the terms $W_{13,\dots,15}$ mix with the terms from the Lagrangian

$$\mathcal{L} = W_{16}\langle \bar{B}\sigma_{\mu\nu}\{\tilde{F}_+^{\mu\nu}, B\}\rangle + W_{17}\langle \bar{B}\sigma_{\mu\nu}[\tilde{F}_+^{\mu\nu}, B]\rangle + W_{18}\langle \bar{B}\sigma_{\mu\nu}B\rangle\langle \tilde{F}_+^{\mu\nu}\rangle, \tag{33}$$

but in both cases the form of the Lagrangian does not change and we can proceed by neglecting the W_6 term and setting $W_1 = W_1^*$ and $W_2 = -\overset{\circ}{M}$ with $\overset{\circ}{M}$ being the baryon mass in the chiral limit [6]. The expansion of the coefficients in terms of the W_i read

$$\begin{aligned} &W_1 = -\frac{1}{2} + \dots, \qquad W_2 = -\overset{\circ}{M}, \\ &W_3 = -\frac{1}{2}D + \dots, \qquad W_4 = -\frac{1}{2}F + \dots, \qquad W_5 = \frac{1}{2}\lambda + \dots, \\ &W_7 = b_D + \dots, \qquad W_8 = b_F + \dots, \qquad W_9 = b_0 + \dots, \\ &W_i = w_i\,(\frac{\sqrt{6}}{F_0}\eta_0 + \bar{\theta}_0) + \dots, \qquad \text{for} \quad i = 10, \dots, 15 \quad , \end{aligned} \tag{34}$$

where the ellipses denote higher orders in $\sqrt{6}\eta_0/F_0 + \bar{\theta}_0$ and we have only shown terms that can contribute to one-loop order. The axial-vector couplings D and F can be determined from semileptonic hyperon decays. A fit to the experimental data delivers $D = 0.80 \pm 0.01$ and $F = 0.46 \pm 0.01$ [12]. The third coupling, λ, is specific to the axial flavor-singlet baryonic current. The coefficients $b_{D,F,0}$ have been determined from the calculation of the baryon masses and the πN σ-term up to fourth chiral order [13] and their mean values are, in units of GeV^{-1},

$$b_D = 0.079, \qquad b_F = -0.316, \qquad b_0 = -0.606. \tag{35}$$

The numerical values of the parameters $w_{10,\dots,15}$ are not known. We leave them undetermined for the time being and will give later an upper bound.

Again the correct vacuum has to be chosen. This is done by setting

$$\begin{aligned} \tilde{\xi}_R &= \sqrt{U_0}\,\xi_R \\ \tilde{\xi}_L &= \sqrt{U_0^\dagger}\,\xi_L, \end{aligned} \tag{36}$$

and we will choose the coset representatives such that

$$\xi_R = \xi_L^\dagger = u = \sqrt{U}. \tag{37}$$

For diagonal external fields v_μ and a_μ one can write

$$\tilde{\Gamma}_\mu = \Gamma_\mu = \frac{1}{2}\Big[u^\dagger\,(\partial_\mu - ir_\mu)\,u \;+\; u\,(\partial_\mu - il_\mu)\,u^\dagger\Big] \tag{38}$$

and

$$\tilde{\xi}_\mu = u_\mu = i\Big[u^\dagger\,(\partial_\mu - ir_\mu)\,u \;-\; u\,(\partial_\mu - il_\mu)\,u^\dagger\Big]. \tag{39}$$

Furthermore, one obtains

$$\begin{aligned} \tilde{\chi}_+ &= \chi_+ - i\mathcal{A}\,(U - U^\dagger) - i\mathcal{B}\chi_- \\ \tilde{\chi}_- &= \chi_- - i\mathcal{A}\,(U + U^\dagger) - i\mathcal{B}\chi_+, \end{aligned} \tag{40}$$

where the quark mass matrix enters in the combinations

$$\chi_\pm = u\chi^\dagger u \pm u^\dagger\chi u^\dagger. \tag{41}$$

Finally, $\tilde{F}^+_{\mu\nu}$ simplifies to

$$\tilde{F}^+_{\mu\nu} = F^+_{\mu\nu} = u^\dagger F^R_{\mu\nu} u + u F^L_{\mu\nu} u^\dagger. \tag{42}$$

The relativistic baryon Lagrangian reads to the order we are working

$$\begin{aligned} \mathcal{L}_{\phi B} \;=\;& i\langle \bar{B}\gamma_\mu [D^\mu, B]\rangle - \overset{\circ}{M}\,\langle \bar{B}B\rangle \\ & -\frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle - \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle + \frac{1}{2}\lambda\langle \bar{B}\gamma_\mu\gamma_5 B\rangle\langle u^\mu\rangle \\ & -ib_D\mathcal{A}\langle \bar{B}\{U - U^\dagger, B\}\rangle - ib_F\mathcal{A}\langle \bar{B}[U - U^\dagger, B]\rangle - ib_0\mathcal{A}\langle \bar{B}B\rangle\langle U - U^\dagger\rangle \\ & +4\mathcal{A}w_{10}\frac{\sqrt{6}}{F_0}\eta_0\langle \bar{B}B\rangle + 6\mathcal{A}w_{12}\frac{\sqrt{6}}{F_0}\eta_0\langle \bar{B}B\rangle \\ & +i(w'_{13}\bar{\theta}_0 + w_{13}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B}\sigma_{\mu\nu}\gamma_5\{F_+^{\mu\nu}, B\}\rangle \\ & +i(w'_{14}\bar{\theta}_0 + w_{14}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B}\sigma_{\mu\nu}\gamma_5[F_+^{\mu\nu}, B]\rangle \\ & +i(w'_{15}\bar{\theta}_0 + w_{15}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B}\sigma_{\mu\nu}\gamma_5 B\rangle\langle F_+^{\mu\nu}\rangle, \end{aligned} \tag{43}$$

where we have neglected meson-baryon interactions with more than one meson field since they do not contribute at one-loop order and terms of $\mathcal{O}(\bar{\theta}_0^2)$ or higher orders are omitted throughout this work. Note that there exist terms of fourth chiral order of the type $\bar{B}\sigma_{\mu\nu}\gamma_5\tilde{\chi}_-\tilde{F}_+^{\mu\nu}B$. Using Eq. (40) for $\tilde{\chi}_-$ they induce CP-violating terms of the form $\bar{\theta}_0\bar{B}\sigma_{\mu\nu}\gamma_5\tilde{F}_+^{\mu\nu}B$ which are already accounted for by the terms $w_{13,14,15}$. This amounts to a renormalization of the couplings $\bar{\theta}_0 w_{13,14,15}$. We have therefore introduced the notation $w'_{13,14,15}$ for these interaction terms, in order to distinguish them from the unrenormalized $w_{13,14,15}$ of the interactions proportional to η_0.

The drawback of the relativistic framework including baryons is that due to the existence of a new mass scale, namely the baryon mass in the chiral limit $\overset{\circ}{M}$, there exists no strict chiral counting scheme, i.e. a one-to-one correspondence between the meson loops and the chiral expansion. In order to overcome this problem one integrates out the heavy degrees of freedom of the baryons, similar to a Foldy-Wouthuysen transformation, so that a chiral counting scheme emerges. To this end, one constructs eigenstates of the velocity projection operator $P_v = (1+\not{v})/2$

$$B_v(x) = e^{i\overset{\circ}{M}\, v\cdot x}\, P_v\, B(x). \tag{44}$$

The Dirac algebra simplifies considerably. It allows to express any Dirac bilinear $\bar{B}_v \Gamma_\mu B_v$ ($\Gamma_\mu = 1, \gamma_\mu, \gamma_5, \ldots$) in terms of the velocity v_μ and the spin operator $2S_\mu = i\gamma_5 \sigma_{\mu\nu} v^\nu$. One can rewrite the Dirac bilinears which appear in the present calculation as

$$\bar{B}_v \gamma_\mu \gamma_5 B_v = 2\bar{B}_v S_\mu B_v, \qquad \bar{B}_v \sigma_{\mu\nu}\gamma_5 B_v = 2i\,(v_\mu \bar{B}_v S_\nu B_v - v_\nu \bar{B}_v S_\mu B_v\,). \tag{45}$$

In the following, we will drop the index v. The Lagrangian of the heavy baryon formulation reads

$$\begin{aligned}
\mathcal{L}_{\phi B} \;=\;& i\langle \bar{B}[v\cdot D, B]\rangle - D\langle \bar{B} S_\mu \{u^\mu, B\}\rangle - F\langle \bar{B} S_\mu [u^\mu, B]\rangle + \lambda \langle \bar{B} S_\mu B\rangle\langle u^\mu\rangle \\
& - i b_D \mathcal{A}\langle \bar{B}\{U - U^\dagger, B\}\rangle - i b_F \mathcal{A}\langle \bar{B}[U - U^\dagger, B]\rangle - i b_0 \mathcal{A}\langle \bar{B}B\rangle\langle U - U^\dagger\rangle \\
& + 4\mathcal{A} w_{10}\frac{\sqrt{6}}{F_0}\eta_0 \langle \bar{B}B\rangle + 6\mathcal{A} w_{12}\frac{\sqrt{6}}{F_0}\eta_0\langle \bar{B}B\rangle \\
& - 4(w'_{13}\bar{\theta}_0 + w_{13}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B} v_\mu S_\nu \{F_+^{\mu\nu}, B\}\rangle \\
& - 4(w'_{14}\bar{\theta}_0 + w_{14}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B} v_\mu S_\nu [F_+^{\mu\nu}, B]\rangle \\
& - 4(w'_{15}\bar{\theta}_0 + w_{15}\frac{\sqrt{6}}{F_0}\eta_0)\langle \bar{B} v_\mu S_\nu B\rangle\langle F_+^{\mu\nu}\rangle.
\end{aligned} \tag{46}$$

No relativistic corrections are needed to the order we are working.

4 The electric dipole moments d_n^γ and d_Λ^γ

In this section, we will calculate the electric dipole moments of the neutron and the Λ at lowest order in chiral perturbation theory, i.e. $\mathcal{O}(p^2)$. Due to the vacuum alignment the baryon Lagrangian contains interaction terms $b_{D,F,0}$ of zeroth chiral order and one-loop diagrams with these vertices will also contribute at $\mathcal{O}(p^2)$. In the relativistic framework of [3] the electric dipole moment of the neutron d_n^γ has been defined via

$$\mathcal{L}_{nEDM} = \frac{1}{2} d_n^\gamma \bar{n} i \sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}, \tag{47}$$

where $F^{\mu\nu}$ is the field strength tensor of the photon field A^μ. We prefer to rewrite this as a form factor

$$D_n^\gamma(q^2)\bar{u}(p')\sigma_{\mu\nu}\gamma_5 u(p) q^\mu \tag{48}$$

with $q = p' - p$ being the momentum transfer. The electric dipole moment is given by

$$d_n^\gamma = D_n^\gamma(q^2 = 0). \tag{49}$$

For the calculation of the form factor in the heavy baryon approach we set $v_\mu = (1, \mathbf{0})$ and use the Breit frame $v \cdot p = v \cdot p'$ since it allows a unique translation of Lorentz-covariant matrix elements into non-relativistic ones. In this frame the form factor reads

$$-2iD_n^\gamma(q^2)\bar{H}v_\nu S \cdot qH + \dots, \tag{50}$$

where H is the large component of u and the ellipsis stands for a similar expression in the small components of u, which is of higher chiral order and will be omitted. The electric dipole moment d_n^γ receives contributions from the w_{13}-term and the loops (we work in the isospin limit $m_u = m_d = \hat{m}$)

$$d_n^\gamma = d_n^{\gamma\,(tree)} + d_n^{\gamma\,(loop)} \tag{51}$$

with

$$d_n^{\gamma\,(tree)} = -8\,e\,\bar{\theta}_0\Big[\frac{1}{3}w_{13}^r + \frac{16}{F_\pi^2 F_0^2 m_{\eta_0}^2}V_3^{(1)}V_0^{(2)}w_{13}\Big], \tag{52}$$

where the pertinent diagrams are shown in Figure 1. Diagram 1b) is missing in [3] since the mesonic Lagrangian used within this work does not have a term linear in the singlet field η_0. Such an interaction originates from the term $V_3\langle\chi_-\rangle$ which is not considered in [3]. The chiral logarithms of the diagrams shown in Fig. 2 read

$$d_n^{\gamma\,(loop)} = \frac{1}{\pi^2 F_\pi^4}eV_0^{(2)}\,\bar{\theta}_0\Big[-(b_D+b_F)(D+F)\ln\frac{m_\pi^2}{\mu^2} + (b_D-b_F)(D-F)\ln\frac{m_K^2}{\mu^2}\Big] \tag{53}$$

with μ being the scale introduced in dimensional regularization. In the Breit frame only diagrams 2a) and b) contribute to the electric dipole moment. Diagrams 2c), d) vanish and 2e), f) are proportional to S_ν and therefore do not contribute to d_n^γ. The loop integral for diagrams 2a) and b) contains also analytic and divergent pieces which can be absorbed by redefining w_{13}'. The divergent pieces of w_{13}' cancel the divergencies from the loops and render the final expression finite. We summarize the remaining analytical contributions in w_{13}^r, so that w_{13}^r in Eq. (52) is understood to be finite. The results in Eqs. (52) and (53) are not in contradiction with the fact that the electric dipole moment induced by the θ_0-term tends to zero if any of the quark masses vanish. If, e.g., $m_u = 0$ then a solution for Eq. (11) is given by $\varphi_u = \theta_0$ and $\varphi_{d,s} = 0$ leading to $\bar{\theta}_0 = \theta_0 - \varphi_u = 0$. Therefore, d_n^γ vanishes in this case.

The results for the Λ are

$$d_\Lambda^{\gamma\,(tree)} = -4\,e\,\bar{\theta}_0\Big[\frac{1}{3}w_{13}^r + \frac{16}{F_\pi^2 F_0^2 m_{\eta_0}^2}V_3^{(1)}V_0^{(2)}w_{13}\Big] \tag{54}$$

and

$$d_\Lambda^{\gamma\,(loop)} = -\frac{1}{\pi^2 F_\pi^4}eV_0^{(2)}\,\bar{\theta}_0\,[b_D F + b_F D]\,\ln\frac{m_K^2}{\mu^2}. \tag{55}$$

Note that the relation $d_n^\gamma = 2d_\Lambda^\gamma$ is only valid at tree level and not for the chiral logarithms as claimed in [3]. The discrepancy is due to the lack of pion loops in the present work. Once one accounts for the mistake made in [3] by replacing $\ln m_\pi$ by $\ln m_K$ in the chiral loop contribution for d_Λ^γ one obtains our result (55).

5 Numerical results and conclusions

In order to compute the numerical results for the electric dipole moments shown in the last section, we use the central values for the parameters $b_D = 0.079\ \text{GeV}^{-1}$, $b_F = -0.316\ \text{GeV}^{-1}$,

$D = 0.80$ and $F = 0.46$. To lowest order in the angles φ_q and using $\hat{m} \ll m_s$ one can express $\bar{\theta}_0$ in terms of θ_0 via

$$\theta_0 \simeq [1 + \frac{8V_0^{(2)}}{F_\pi^2 m_\pi^2}]\bar{\theta}_0. \tag{56}$$

From the calculation of the η and η' masses and decay constants one can extract the value for $V_0^{(2)}$ [14]

$$V_0^{(2)} \simeq -\frac{27}{4} F_\pi^4 \simeq -5.0 \times 10^{-4}\ \mathrm{GeV}^4, \tag{57}$$

so that

$$\bar{\theta}_0 \simeq \frac{F_\pi^2 m_\pi^2}{8V_0^{(2)}}\, \theta_0 \simeq -0.04\, \theta_0. \tag{58}$$

Inserting this into the loop contribution and using $\mu = 1$ GeV, $m_{\eta_0} \simeq m_{\eta'} = 958$ MeV we obtain

$$\begin{aligned} d_n^{\gamma\,(loop)} &= -7.5 \times 10^{-16}\, \theta_0\, e\,\mathrm{cm} \\ d_\Lambda^{\gamma\,(loop)} &= -1.7 \times 10^{-16}\, \theta_0\, e\,\mathrm{cm}. \end{aligned} \tag{59}$$

The numerical result for $d_n^{\gamma\,(loop)}$ is in agreement with the one given in [3] once one accounts for the different values of the parameters b_D, b_F, D, F and the scale μ used within that work. The result for the chiral logarithm of $d_\Lambda^{\gamma\,(loop)}$ is considerably smaller than for $d_n^{\gamma\,(loop)}$ since there is no contribution from the pion loops which dominate in the case of d_n^γ.

A precise numerical value for the tree contribution to the electric dipole moments cannot be given since the parameters w_{13} and w_{13}^r are not known. However, we will give an upper bound for their contribution based on large N_c arguments which seem to work well in the purely mesonic sector [14, 15]. In the present investigation we are only interested in an order of magnitude estimate for the vacuum angle θ and for this purpose it is sufficient to give a numerical range for the tree level contribution. We will first estimate the ratio of diagrams 1a) and 1b) using $1/N_c$ arguments. Applying large N_c counting rules, see e.g. [9, 10], both w_{13} and w_{13}^r are of order $\mathcal{O}(N_c^0)$ so that we can assume $|w_{13}/w_{13}^r| = \mathcal{O}(1)$. One obtains the ratio

$$\frac{|d_n^{\gamma\,(1b)}|}{|d_n^{\gamma\,(1a)}|} = \frac{|d_\Lambda^{\gamma\,(1b)}|}{|d_\Lambda^{\gamma\,(1a)}|} \simeq \frac{48}{F_\pi^4 m_{\eta'}^2} |V_0^{(2)} V_3^{(1)}| \simeq 0.12, \tag{60}$$

where we have used $F_\pi/F_0 = 1 + \mathcal{O}(N_c^{-1})$ and taken the value for $V_3^{(1)}$ from [14]

$$V_3^{(1)} \simeq 0.04 F_\pi^2 \simeq 3.5 \times 10^{-4}\ \mathrm{GeV}^2. \tag{61}$$

Diagram 1b) turns out to be insignificant in our estimate.

Based on large N_c arguments one can also give an upper bound for w_{13}^r. The contact terms of the Lagrangian in Eq. (33), which contribute to the magnetic moments of the baryons, describe – similar to the w_{13}^r-term – the coupling of the field strength tensor $F_{\mu\nu}^+$ to the baryons. But the leading coefficient of the Taylor expansion of W_{16}, w_{16}, is of order $\mathcal{O}(N_c^1)$, whereas $|w_{13}^r| \simeq |w_{13}| = \mathcal{O}(N_c^0)$, so that we can assume $|w_{13}^{(r)}| < |w_{16}|$. In a calculation of the baryon magnetic moments to fourth chiral order w_{16} has been determined to be [16]

$$w_{16} \simeq 0.4\ \mathrm{GeV}^{-1}. \tag{62}$$

Inserting this upper limit for w_{13}^r one obtains

$$\begin{aligned} |d_n^{\gamma\,(tree)}| &< 9.6 \times 10^{-16}\,\theta_0\, e\,\mathrm{cm} \\ |d_\Lambda^{\gamma\,(tree)}| &< 4.8 \times 10^{-16}\,\theta_0\, e\,\mathrm{cm}. \end{aligned} \tag{63}$$

Since we have taken quite conservative limits, the tree level contributions could be dominating if the extreme values are chosen. A more realistic estimate might be obtained by setting $|w_{13}^{(r)}/w_{16}| \simeq \frac{1}{3}$ for $N_c = 3$, so that

$$\begin{aligned} |d_n^{\gamma\,(tree)}| &\simeq 3.2 \times 10^{-16}\,\theta_0\, e\,\mathrm{cm} \\ |d_\Lambda^{\gamma\,(tree)}| &\simeq 1.6 \times 10^{-16}\,\theta_0\, e\,\mathrm{cm}. \end{aligned} \tag{64}$$

While the chiral logarithms dominate for d_n^γ, the tree level contribution can be substantial for the Λ. This leads to

$$\begin{aligned} d_n^\gamma &= (-7.5 \pm 3.2) \times 10^{-16}\,\theta_0\, e\,\mathrm{cm} \\ d_\Lambda^\gamma &= (-1.7 \pm 1.6) \times 10^{-16}\,\theta_0\, e\,\mathrm{cm}, \end{aligned} \tag{65}$$

where the theoretical uncertainty is given by the estimate of the tree level contribution in Eq. (64). From a comparison of the central value for d_n^γ with the experimental upper limit in Eq. (1) we obtain

$$|\theta_0| < 8.4 \times 10^{-11}. \tag{66}$$

In the case of the Λ the experimental constraint is given by

$$d_\Lambda^\gamma < 1.5 \times 10^{-16}\, e\,\mathrm{cm} \tag{67}$$

which leads to

$$|\theta_0| < 0.9. \tag{68}$$

We have shown that it is possible to obtain a reliable limit for the vacuum angle θ_0 by calculating the electric dipole moment of the neutron within the framework of heavy baryon chiral perturbation theory. To this end, we have constructed the most general effective Lagrangian up to one-loop order in the presence of the vacuum angle θ_0 with the method proposed in [6]. The theoretical uncertainty from unknown parameters at tree level has been estimated by using large N_c arguments. While the chiral loops dominate for the electric dipole moment of the neutron, the counterterms can be substantial in the case of the Λ.

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Figure captions

Fig.1 Shown are the tree diagrams for the electric dipole moment. Solid and dashed lines denote baryons and pseudoscalar mesons, respectively. The wavy line represents a photon and the dot is a CP-violating vertex.

Fig.2 Loop diagrams contributing to the electric dipole moment. Solid and dashed lines denote baryons and pseudoscalar mesons, respectively. The wavy line represents a photon and the dot is a CP-violating vertex.

Figure 1

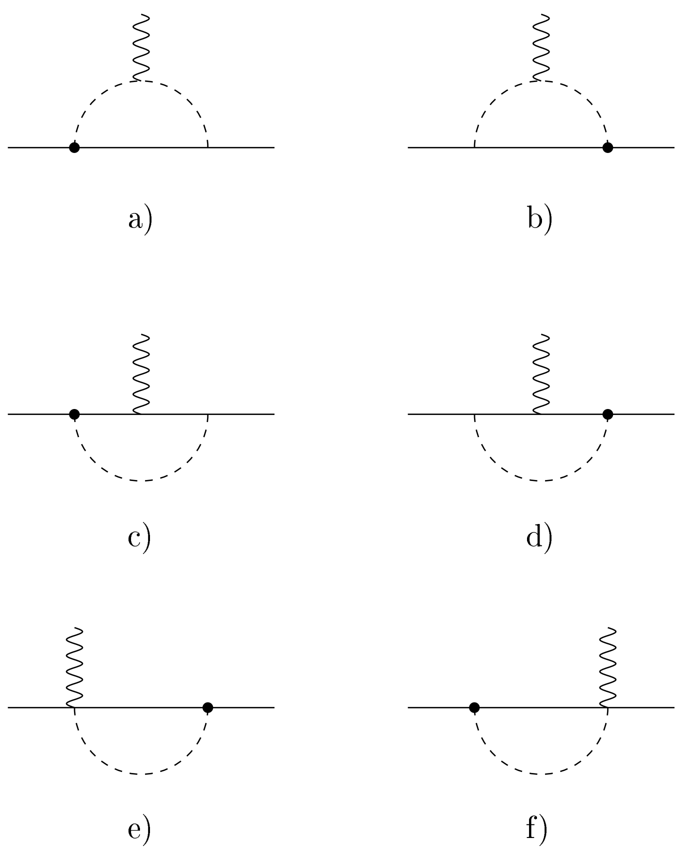

Figure 2